(6 pages) Reg. No. : ....

Code No. : 20293 E **Sub. Code : AMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. In $[0, 1]$ with usual metric, $B\left(0, \frac{1}{4}\right)$ is ————.

   (a) $\left(-\frac{1}{4}, \frac{1}{4}\right)$ (b) $\left[0, \frac{1}{4}\right]$

   (c) $\left[0, \frac{1}{4}\right)$ (d) $\left(0, \frac{1}{4}\right)$

2. Which of the following subsets of R is not open?

   (a) $(0, 1)$ (b) ϕ

   (c) $(1, 2)\cup(3, 4)$ (d) Q

3. $f : M_1 \to M_2$ is continuous if and only if

   (a) $x_n - x = 0 \Rightarrow f(x_n) - f(x) = 0$

   (b) $x_n \to x \Rightarrow f(x_n) = f(x)$

   (c) $(x_n) \to x \Rightarrow (f(x_n)) \to f(x)$

   (d) $x_n - x \to 0 \Rightarrow f(x_n - x) \to 0$

4. The function $f : (0, 1) \to R$ defined by $f(x) = \frac{1}{x}$ is

   (a) not continuous

   (b) uniformly continuous

   (c) not uniformly continuous

   (d) neither continuous nor uniformly continuous

5. If $A = (0, 1] \subseteq R$, then $\overline{A}$ is ————.

   (a) $(0, 1)$ (b) $[0, 1]$

   (c) $(0, 1]$ (d) $[0,1)$

6. A connected subset of R is

   (a) $[4, 7]\cup[8, 10]$ (b) $[4, 6]\cup[5,7]$

   (c) $[4, 7)\cup(7, 8)$ (d) Q

7. $\bigcup_{n=1}^{\infty}[0, n) = ?$

   (a) $[0,\infty]$ (b) $(0,\infty)$

   (c) $[0, \infty)$ (d) $(0,\infty]$

8. A compact subset of R is ————.

   (a) $[0, \infty)$ (b) $(3, 4)$

   (c) Q (d) $[1, 2.8]$

9. $\bigcup_{n=1}^{\infty}\left(0, \frac{1}{n}\right) = ?$

   (a) $(0,1)$ (b) ϕ

   (c) $\{0\}$ (d) $(0, 1]$

10. In $R \times R$, $\overline{Q \times Q}$ is ————.

    (a) ϕ (b) Q^2

    (c) $R \times R$ (d) $Z \times Z$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In any metric space prove that each open ball is an open set.

    Or

    (b) Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

12. (a) Show that the function $f : R \to R$ defined by

    $$f(x) = \begin{cases} 0, & \text{if } x \text{ is irrational} \\ 1, & \text{if } x \text{ is rational} \end{cases}$$

    is not continuous.

    Or

    (b) Prove that $f : M_1 \to M_2$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq M_1$.

13. (a) If A is a connected subset of the metric space M. Prove that $\overline{A}$ is connected.

    Or

    (b) Show that the continuous image of a connected metric space is connected.

14. (a) Prove that continuous image of a compact metric space is compact.

    Or

    (b) If A is a compact subset of a metric space (M,d), prove that A is closed.

15. (a) Let A be a subset of a metric space M. If A is totally bounded, show that A is bounded.

Or

(b) Show that a metric space is compact if and only if any family of closed sets with finite intersection property has non empty intersection.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Cantor's interaction theorem.

Or

(b) State and prove Baire's category theorem.

17. (a) (i) Let (M, d) be a metric space. Let $a \in M$, show that the function $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

(ii) Let (M, d) be any metric space. Let $f : M \to R$, $g : M \to R$ be two continuous functions. Prove that $f + g$ is continuous.

Or

(b) Prove that $f : R \to R$ is continuous at $a \in R$ f and only if $w(f, a) = 0$.

18. (a) Prove that R is a connected metric space.

Or

(b) (i) If A and B are connected subsets of a metric space M and $A \cap B = \phi$. Prove that $A \cup B$ is a connected set.

(ii) State and prove the Intermediate value theorem.

19. (a) State and prove Heine Borel Theorem.

Or

(b) Let (M_1, d_1) be a compact metric space and (M_2, d_2), be any metric space. If $f : M_1 \to M_2$ is continuous, prove that f is uniformly continuous on M.

20. (a) If A is a totally bounded set. Prove that $\overline{A}$ is also totally bounded.

Or

(b) Prove that the metric space M is compact iff any family $\{A_\alpha\}$ of closed sets with finite intersection property has non empty intersection.
